

## СИММЕТРИЯ И СИММЕТРИЧНЫЕ ПРЕДИКАТЫ SYMMETRY AND SYMMETRICAL PREDICATES\*

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Цель этой статьи – проанализировать разницу между математическими определениями симметрии и понятием симметрии, которое бы наилучшим образом соответствовало лингвистическим обобщениям. Это требует тщательного анализа лингвистического «поведения» симметричных и несимметричных предикатов.

### 1. Background

“Symmetrical predicates” have distinctive linguistic properties in many languages. But the concept of “symmetry” merits closer examination, especially in the light of the controversial claim by the psychologist Amos Tversky [1] that the concept ‘similar’, a standard example of a symmetrical predicate, is in fact not symmetrical. Tversky’s evidence includes the fact that experimental subjects generally rate (1a) as holding to a higher degree than (1b).

- (1) a. *North Korea is similar to Red China.*  
b. *Red China is similar to North Korea.*

Lila Gleitman and colleagues argue in an interesting paper [2] that ‘similar’ is symmetrical, and that the difference in judgments reflects the independent contribution of figure-ground differences encoded in the syntax. They argue in support of a robust linguistic distinction between symmetrical and “asymmetrical” predicates. Gleitman *et al* use a semantic paraphrase test as a central property in characterizing linguistically symmetrical predicates in English: does the *intransitive* version of a given predicate have a meaning close to the meaning of an overt reciprocal with the corresponding *transitive* version? This test is illustrated in (2) and (3) below, where (2a) and (2b), with symmetrical *meet*, are close in meaning, but (3a) and (3b), with the “asymmetrical” *drown*, are not.

- (2) a. *John and Bill meet.*  
b. *John and Bill meet each other.*
- (3) a. *John and Bill drown.*  
b. *John and Bill drown each other.*

Gleitman *et al*’s paper analyzes symmetrical and what I will call “quasi-symmetrical” or “sometimes-symmetrical” predicates in English, including verbs (*meet, kiss*), and adjectives (*similar*), to which I will add nouns (*sibling, brother*). Their paper addresses and solves the mysteries raised by Tversky’s work concerning the apparent non-symmetrical behavior of symmetrical predicates like *similar*.

The arguments in the paper are convincing; at the same time, Gleitman *et al*’s uses of the terms “symmetrical” and “asymmetrical” do not always fit the standard mathematical definitions, given in (4). (Variation in definitions is discussed in Section 2.)

- (4) a. A relation R is symmetrical iff for all x, y: if R(x,y), then R(y,x).  
b. A relation R is asymmetrical iff for all x, y: if R(x,y), then  $\neg$  R(y,x).  
c. A relation R is non-symmetrical iff it is not symmetrical.

One goal of this paper is to analyze the differences between the standard mathematical definitions of symmetry and a concept of symmetry that would fit best with observed linguistic generalizations. In order to try to modify the mathematical definitions to better fit the linguistic facts, we have to look at the linguistic facts more carefully as well.

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## 2. Definitions and examples

### 2.1. The standard terminology and its consequences, with examples

The definitions of *symmetrical*, *asymmetrical*, *non-symmetrical* in (4) above are taken from Partee, ter Meulen, and Wall [3] (PtMW) (where the equivalent *symmetric*, *asymmetric*, *non-symmetric* are used), which in turn followed such classics as [4]. There is a fourth term in this family, *antisymmetrical*, defined in [3] and elsewhere as follows:

(5) A relation  $R$  is *antisymmetrical* iff for all  $x, y$ : if  $R(x,y)$  and  $R(y,x)$ , then  $x = y$ .

Typical examples:  $\leq$  is antisymmetrical, whereas  $<$  is asymmetrical. An *antisymmetrical* relation is asymmetrical except for possible instances of  $xRx$ . Turning it around, an *asymmetrical* relation is one that is antisymmetrical and irreflexive.

We begin by reviewing some consequences of the standard definitions and some examples.

First of all: Are the four properties mutually exclusive? No.

(i) Every asymmetrical relation is non-symmetrical. One can call *asymmetrical* a ‘strong’ negation of *symmetrical*: it means “never symmetrical”. *Non-symmetrical* just says “sometimes not”:  $R$  is non-symmetrical iff it is not symmetrical.

(ii) Every asymmetrical relation is also antisymmetrical. If it’s asymmetrical, the if-clause in the definition of *antisymmetrical* is never satisfied, so the entire statement is vacuously satisfied.

(iii) Can a symmetrical relation ever have any of the *a-*, *non-*, *anti-* symmetry properties? Yes: the empty relation, for instance, is symmetrical, asymmetrical, and antisymmetrical. And the identity relation on some domain  $D$ , which has all and only pairs of the form  $\langle a,a \rangle$  for a in  $D$ , is both symmetrical and antisymmetrical.

Secondly, are the four properties exhaustive? That is, does every relation have at least one of those properties? Yes, because *symmetrical* and *non-symmetrical* are complements: every relation on a given domain  $A \times B$  must be either symmetrical or non-symmetrical.

The relation *father of* is asymmetrical, since for all  $x, y$ , if  $x$  is the father of  $y$ , then  $y$  is not the father of  $x$ .

The relation *sibling of* (i.e. *brother or sister of*) is symmetrical, since for all  $x, y$ , if  $x$  is a sibling of  $y$ , then  $y$  is a sibling of  $x$ .

What about the relation *brother of*: is it true that for all  $x,y$ , if  $x$  is the brother of  $y$ , then  $y$  is the brother of  $x$ ? This can’t be answered without specifying the **domain** of the relation.

(a) On the domain of all humans, *brother of* is neither symmetrical nor asymmetrical, but non-symmetrical: if  $x$  is brother of  $y$ ,  $y$  may be brother of  $x$  or sister of  $x$ .

(b) On the domain of all male humans, *brother of* is symmetrical.

We’ll return to this case in Section 4.2, because *brother* behaves linguistically like a symmetrical predicate in sentences like (6).

(6) *John and Bill are brothers.*

### 2.2. Colloquial terminology

**The terminology:** When talking about the *brother-of* relation, students often say “sometimes it’s symmetrical and sometimes it’s asymmetrical.” Or “it has both symmetrical and asymmetrical instances.” But according to the definitions, *symmetrical* and *asymmetrical* are properties of *relations*, and it doesn’t make sense to apply those terms to *instances*, or to say that a relation has such a property “sometimes”. But it’s clear what the students mean, and we can give new definitions to extend the terminology in these ways.

**Defining “symmetrical instances”:**

(7) (a) “Sometimes *brother* is symmetrical” and “*Brother* has symmetrical instances” can both be defined as follows: there are pairs  $a,b$  such that  $a$  is brother of  $b$  and  $b$  is brother of  $a$ .

(b) *Brother* has asymmetrical instances” can be similarly defined as saying that there are pairs  $a,b$  such that  $a$  is brother of  $b$  and  $b$  is not brother of  $a$ .

Note that in both cases we start from an instance of  $aRb$ , and then ask whether  $bRa$  holds. Since either it does

or it doesn't, there are only two cases, symmetric or asymmetric, for 'instances'.

Given this notion of symmetrical and asymmetrical *instances*, the original concept of a symmetrical *relation* becomes "a relation that has no asymmetrical instances" (this negative characterization takes care of the *if-then* nature of the original definition, and is consistent with the fact that the empty relation is symmetrical). An asymmetrical relation becomes "a relation that has no symmetrical instances". And a non-symmetrical relation becomes "a relation that has at least one asymmetrical instance."

**Defining "sometimes symmetrical and sometimes asymmetrical":**

- (8) The colloquial locution "On the domain  $H \times H$  of all humans, sometimes *brother* is symmetrical and sometimes it's asymmetrical" can be defined as "The domain  $H \times H$  can be partitioned into two non-empty subdomains such that *brother* is symmetrical on one and asymmetrical on the other."

If we let  $F$  be the set of female humans and  $M$  be the set of males, *brother* is symmetrical on  $M \times M$  and asymmetrical on  $M \times F \cup F \times M$ . (This isn't yet a partition of  $H \times H$ , because we haven't included  $F \times F$ . But since *brother* is the empty relation on  $F \times F$ , it is both symmetrical and asymmetrical on that domain, so we could add  $F \times F$  to either cell of the partition without changing the result.)

**2.3. Alternative terminology**

Not all texts use the terms *asymmetrical* and *non-symmetrical* in the way defined in (4).

- (i) Gleitman *et al* use **asymmetrical** in the way that I defined *non-symmetrical*. Initially I thought their usage was a mistake, but I have discovered that both usages are common. Authors differ as to which term carries the meaning "not symmetrical".

I will continue to use *asymmetrical* as defined in (4), but it's important to be aware of the way Gleitman *et al* are using it, and the fact that that is also an accepted usage.

- (ii) There are also two definitions of **non-symmetrical** in the literature.
- (a) The definition in (4), "not symmetrical", can be called the "broad sense" of 'non-symmetrical'.
  - (b) The other, "narrow sense" defines non-symmetrical as "neither symmetrical nor asymmetrical".

There are advantages and disadvantages to both notions. The first choice is more intuitive and linguistically uniform: «non» means «not», and that holds also for *non-reflexive* and *non-transitive*. The second choice but not the first provides convenient terminology for a three-way partition of relations (on a specified domain) into symmetrical, asymmetrical, and neither.

**3. What do "symmetrical", "non-symmetrical" apply to?**

**3.1. The "asymmetrical act of drowning someone".**

In standard mathematical terminology, the properties "symmetrical" and "asymmetrical" apply to **binary relations** and nothing else. But in other parts of mathematics the properties may apply to geometrical figures, to certain algebraic structures, and to other mathematical objects. And in everyday English and in Gleitman *et al*'s article, those expressions have additional uses relating to acts, propositions, situations, as illustrated in (9).

- (9) a. *asymmetric warfare*: warfare between two very unequal forces, with the two sides often using very different methods.
- b. *an asymmetric power situation*: a situation involving two individuals or two states  $a$  and  $b$  in which  $a$  has much more power over  $b$  than  $b$  has over  $a$ .
- c. *asymmetric interdependence*: a state of interdependence between two (or more) individuals or other entities, in which there are pairs  $a, b$  such that  $a$  is much more dependent on  $b$  than  $b$  is on  $a$ .

Gleitman *et al* begin their discussion of the relation between reciprocal structures with *each other* and reciprocal interpretations of intransitive symmetrical and non-symmetrical predicates, as in (2) and (3), with the observation that many obviously non-symmetrical predicates license the reciprocal construction. "Thus if John drowns Bill while Bill drowns John, we can say

- (10) *John and Bill drown each other.*" [2, p.326]

They describe (10) as depicting a situation in which two individuals reciprocate the “decidedly asymmetric act of drowning someone.” What is an “asymmetric act”?

### 3.2. In what sense is drowning an “asymmetric act”?

I can see three possible factors behind the idea that drowning is an “asymmetric act.”

(i) In most cases where a drowns b, b does not drown a. If we make symmetry a graded notion with formal asymmetry and formal symmetry as the extremes, and rank the non-symmetrical relations on a scale according to the proportion of aRb cases for which bRa also holds, then the relation aRb expressed by “a drowns b” is very close to the asymmetrical end of the scale. (Similarly, ‘is a friend of’ and ‘is best friend of’ may be non-symmetrical but close to the symmetrical end.)

(ii) We may be influenced by **geometrical symmetry**/asymmetry when we *picture* an act. Most imaginable ways in which someone drowns someone have the agent and victim in different positions – it’s not a visually symmetrical picture. (But one can imagine a possible double murder where they manage to drown each other in a symmetrical-looking way.)

There is the related linguistic factor of Agent and Patient roles and how different they are. This is discussed by Gleitman *et al*, but in a different context, together with Figure-Ground changes, as involving changes in something other than actual semantic content. But semantics may be relevant: for transitive *drown*, the Agent and Patient roles are much more distinct than for, say *meet*, and there are no Agent or Patient roles at all with adjectival predicates like *similar* or noun predicates like *sibling* or *uncle* or prepositions like *near*.

(iii) The role of the **event argument** [5, 6] may be relevant. Active verbs have an event argument, which may not be present with adjectives or stative predicates. Suppose the basic argument structure of *drown* is as in (11).

(11) *drown(e,a,b)* : *e* is an event of *a* drowning *b*.

Then the question arises: can one and the same event be an event of a drowning b and b drowning a? This is a non-trivial question: compare debates about whether a buying event and a corresponding selling event are always/sometimes/never the same event. If the answer is “no”, that is, if one can argue that they are never the same event even if they happen at the same time, that could be another basis for the intuition that drowning is an asymmetrical act.

### 3.3. What is the relation between ‘drowned’ and ‘drowned each other’?

One of Gleitman *et al*’s insights is that a reciprocal sentence can in some sense turn an *asymmetrical* relation into a *non-symmetrical* one, i.e. can allow ‘symmetrical instances’ even if the core lexical predicate does not. How can we make formal sense of this intuitively appealing idea?

Consider sentence (10) again, here converted to the past tense for naturalness.

(10) *John and Bill drowned each other.*

And consider again Gleitman *et al*’s statement that “In (10), these two reprobates have reciprocated the decidedly asymmetric act of drowning someone.” We have considered three possible bases for calling the act of drowning asymmetric. What might it mean then to say that John and Bill have “reciprocated” an asymmetric act?

First we note that the overt reciprocal construction certainly allows for two distinct events, as illustrated by the possibility of attributing different properties to the two events, as in (12).

(12) *John and Bill drowned each other by different methods.*

For a happier verb, and one for which it’s easier to imagine two completely separate events, consider *rescue* in (13).

(13) *John and Bill rescued each other by different methods/ on different days.*

And as Gleitman *et al* note, even for a (relatively) symmetrical predicate like *kiss*, the reciprocal construction in (14a) allows for two distinct acts, unlike the intransitive (14b).

(14)a. *John and Mary kissed each other on different parts of their faces.*

b. *John and Mary kissed \*on different parts of their faces.* [OK on an irrelevant iterative reading – many (mutual) kisses, in various places.]

Returning to *drown*: Suppose that the meaning postulate in (15) is correct.

(15)  $drown'(e,a,b) \rightarrow \neg drown'(e,b,a)$  for all  $a,b$  such that  $a \neq b$ .

This meaning postulate is a way of saying that the lexical *drown* relation is antisymmetrical relative to a fixed event  $e$ . (It's antisymmetrical, not asymmetrical, since a person can drown himself.)

Even if lexical *drown* is thus antisymmetrical, the sentence 'John drowned Bill' is consistent with the sentence 'Bill drowned John', because in a full sentence, the event argument is existentially quantified, as in (16).

(16) Possible: both  $\exists e_1(drown'(e_1, a, b))$  and  $\exists e_2(drown'(e_2, b, a))$

In other words, "John drowned Bill" and "Bill drowned John" can both be true by virtue of two different events, since each just says that *there is an event* of the given kind.

So if the sentence 'John drowned Bill' includes an existentially quantified event argument, that explains how the sentence can express a merely non-symmetrical relation, even if the core predicate *drown* inside it is asymmetrical or antisymmetrical. Thus we can formalize Gleitman *et al*'s idea that the reciprocal construction can turn an asymmetrical relation into a non-symmetrical one, one that can have "symmetrical instances."

#### 4. Non-symmetrical predicates that pass the linguistic test for "symmetrical predicates"

Predicates of various syntactic classes that are classed as symmetrical predicates by Gleitman *et al* include the verbs *meet*, *kiss*, the adjective *similar*, and I will add the noun *sibling*.

The *linguistic* property considered as central by Gleitman *et al* for distinguishing the symmetric from the non-symmetric class is the test with *intransitives*, whether their meaning is or is not similar to the meaning of an overt reciprocal, as discussed in Section 1.

##### 4.1. The observed meaning difference between reciprocal and plural intransitives with symmetric predicates.

Gleitman *et al* note carefully that even in the case of most of the predicates they class as symmetrical, such as *kiss*, the meanings of the intransitive-plural and the reciprocal sentences are not judged identical, only similar. This observed difference may also be explainable on the basis of the event argument plus the lexical semantics of the derived intransitive.

- (17) a. *Susan and Bill kissed each other.*  
 b. *Susan and Bill kissed.*

(i) The adverbial test illustrated with (14a-b) supports the idea that there is just one event in the intransitive sentence, two in the overtly reciprocal sentence. But Gleitman *et al* raise the interesting question of how the structure and compositional semantic derivation of (17b) differ from that of a sentence with an intransitive variant of a non-symmetrical verb, such as *drown*, as in (3a), *John and Bill drown*.

(ii) First of all, let us grant that (17b) has a single event argument and describes the occurrence of a common event with two participants, and let us look at the structure of (17a). Appealing to analyses such as those in [7], [8], [9], we may argue that the overt reciprocal sentence (17a), while not derived from a conjunction of two sentences (we agree with Gleitman *et al* about that), nevertheless describes a non-atomic event, one that has parts that are subevents of the given event.

(iii) Returning to the intransitive (17b), I would argue that if it unambiguously posits a single event of mutual kissing, then the intransitive verb *kiss* needs to be considered a separate derived variant of the transitive *kiss*. Lexical rules for deriving several kinds of intransitive alternants of transitive verbs were proposed by Dowty [10]. Russian marks several sorts of derived intransitives (reciprocals, reflexives, unaccusatives) with the morpheme *sja*. English uses "zero-derivation" instead. The following is based on Dowty's rule, modified to include an event argument.

(18) transitive *kiss*:  $\lambda y \lambda x \lambda e \text{ kiss}'(e,x,y)$

inherently reciprocal  $kiss_{\text{recip}}$ :  $\lambda X \lambda E [\forall y \leq_i X. \exists z \leq_j X. \exists e \in E . \text{ kiss}'(e,y,z)]$

This is to be read as follows:  $kiss_{\text{recip}}$ , the derived 'inherently reciprocal' version of *kiss*, is predicated of a single 'plural event'  $E$  and a single plural entity  $X$ , and  $kiss_{\text{recip}}(E,X)$  says that for every atomic individual  $y$  who is part of  $X$ , there is a subevent  $e$  of  $E$  and an atomic individual  $z$  that is part of  $X$  such that  $e$  is an event of  $y$  kissing  $z$ .

It is likely that intransitive *kiss* has further lexicalized and has a more specific meaning than that, perhaps requiring that the ones who are kissing are not just kissing each other but kissing each other on the mouth. Dowty's lexical

rules are proposed as semi-productive sense-deriving processes which can be followed by further lexicalization. The meaning suggested in (18) is a simple version of a reciprocal meaning; see the works mentioned above for more sophisticated suggestions for the semantics of reciprocals.

What is crucial is that it is possible to say that there is a single event with a single plural actant in the case of the intransitives, while characterizing that event in terms of subevents involving the corresponding transitive predicates.

#### **4.2. Why does *brother* pattern as a symmetrical predicate?**

We observed earlier that *brother* is non-symmetrical on the class of humans, and symmetrical only when restricted to the domain of males. Yet it patterns with symmetrical predicates in forming plurals as in (6). This seems to challenge one of the main conclusions of Gleitman *et al*, that “overwhelmingly often, symmetrical concepts are expressed by predicates marked with a special lexical feature. This lexical feature licenses a reading of noun phrase conjunction to express reciprocity of the relation between the nominal conjuncts. No asymmetrical [= non-symmetrical] concepts have this feature.” (p. 365.)

*Asymmetrical* is used in that paper as I use *non-symmetrical* (perhaps limited to non-symmetricals that are not “nearly symmetrical”). *Brother* is clearly non-symmetrical, since only about half the instances are ‘symmetrical instances’. And yet it patterns like the fully symmetrical *sibling* and *cousin* and the nearly symmetrical *friend*, and unlike the nearly asymmetrical *uncle*.

Why does this happen with *brother* (and *sister*)? Is *brother* on the domain of males a separate concept? Is there a separate symmetrical lexical item *brother* distinct from the non-symmetrical one<sup>1</sup>? Or is it rather that with nouns, pluralization is able to pick out the symmetrical subrelation of the broader relation? I don’t have an answer. One place I would look for ideas is in the work of Staroverov on conjoined relational nouns of the type *husband and wife*, *brother and sister* [11], since there is something intrinsically reciprocal about those predicates, and they are very close in meaning to *spouses*, *siblings* respectively.

### **5. Conclusions**

A great deal of progress in semantics has emerged from studying areas where certain ‘standard’ logical notions do not seem to have a perfect fit with their nearest natural language equivalents, and finding better notions where they turn out to be needed. What I have argued in this paper is that there is an interesting field for research starting from apparent mismatches between logicians’ definitions of symmetry and asymmetry and the way those notions are used in ordinary language, a field of research whose value is clear from the very fruitful insights in the pioneering work of Gleitman *et al*.

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<sup>1</sup> Some of the participants in the 2007 colloquium at the University of Pennsylvania informed me that there are languages which have separate lexemes for ‘brother [of a male]’ and ‘brother [of a female]’, suggesting that the symmetrical subrelation is indeed a natural and salient concept.